2 Differential calculus

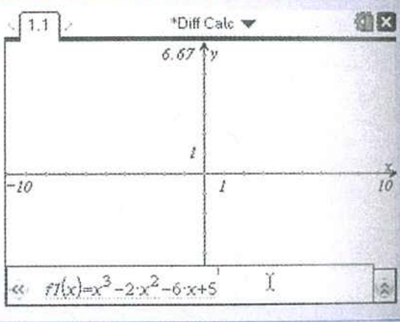
Finding gradients, tangents and maximum and minimum points

2.1 Finding the gradient at a point

Example 21

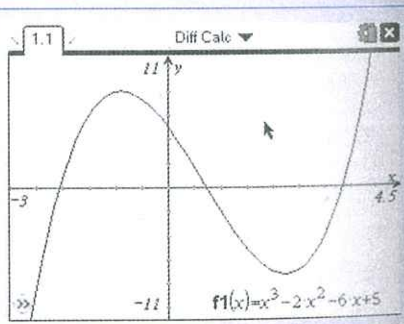
Find the gradient of the cubic function $y = x^3 - 2x^2 - 6x + 5$

Open a new document and add a Graphs page.
The entry line is displayed at the bottom of the work area.
The default graph type is Function, so the form '$f1(x)=$' is displayed.
The default axes are $-10 \leq x \leq 10$ and $-6.67 \leq y \leq 6.67$.
Type $x^3 - 2x^2 - 6x + 5$ and press enter.
(**Note:** Type × ^ 3 ▶ to enter x^3. The ▶ returns you to the baseline from the exponent.)

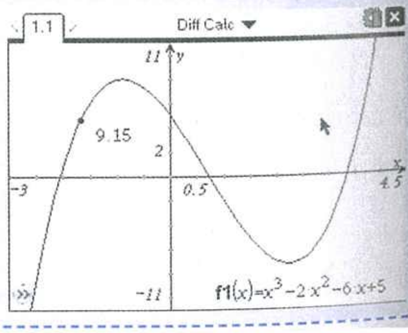


Pan the axes to get a better view of the curve and then grab the x- and y-axes to fit the curve to the window.

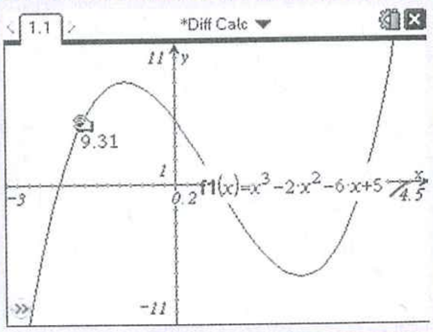
For help with panning and changing axes, see your GDC manual.



Press menu 6:Analyze Graph | 5: $\frac{dy}{dx}$
Press enter
Using the touchpad, move the hand icon towards the curve. As it approaches the curve, it turns to a pencil icon and displays the numerical value of the gradient.
Press enter to attach a point on the curve.



Use the touchpad to move the hand icon to the point.
You can move the point along the curve and observe how the gradient changes as the point moves.
Here, gradient at point = 9.31.

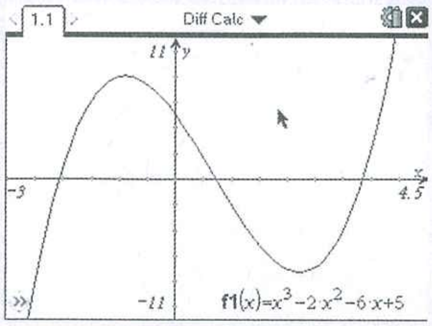


2.2 Drawing a tangent to a curve

Example 22

Draw a tangent to the curve $y = x^3 - 2x^2 - 6x + 5$

First draw the graph of $y = x^3 - 2x^2 - 6x + 5$ (see Example 21).



Press menu 7:Points & Lines | 7:Tangent

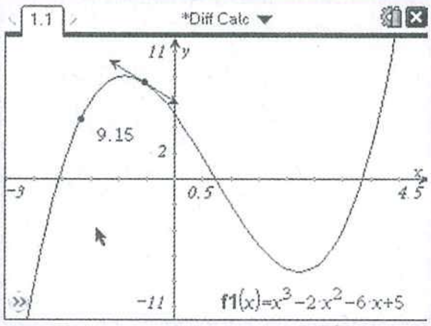
Press enter

Using the touchpad, move the arrow cursor towards the curve. As it approaches the curve, it turns to a hand cursor.

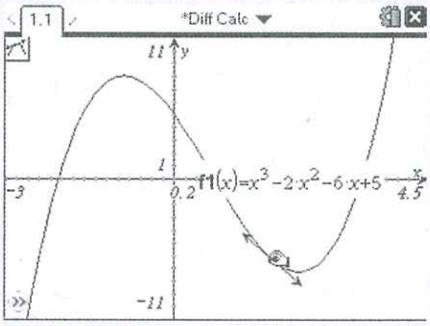
Press enter

The cursor changes to a pencil and displays 'point on'.

Choose a point where you want to draw a tangent and press enter.



You can move the point that the tangent line is attached to with the touchpad.



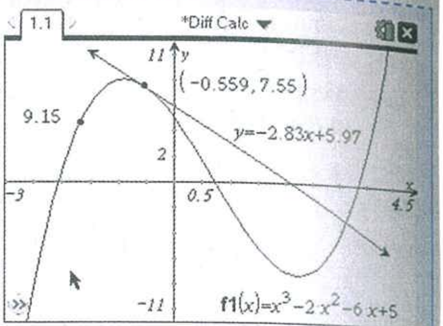
Use the touchpad to drag the arrows at each end of the tangent line to extend it.

Press ctrl menu with the tangent line selected – move to the arrow at the end and look for the word 'line'.

Choose 7:Coordinates and Equations

Click on the line to display the equation of the tangent: $y = -2.83x + 5.97$.

Click on the point to display the coordinates of the point: $(-0.559, 7.55)$.

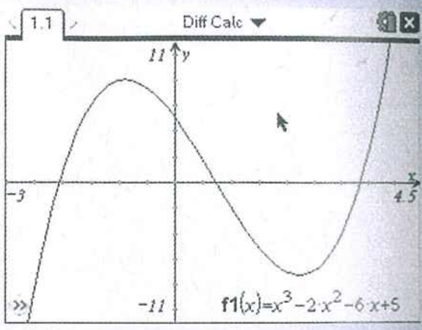


2.3 Finding maximum and minimum points

Example 23

Find the local maximum and local minimum points on the cubic curve:
$y = x^3 - 2x^2 - 6x + 5$

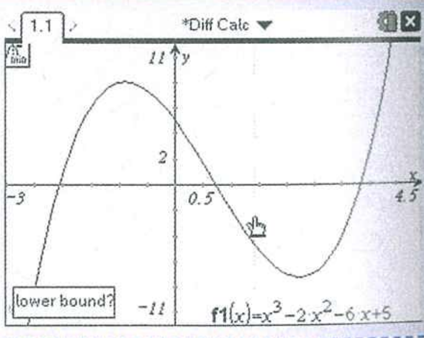
First draw the graph of $y = x^3 - 2x^2 - 6x + 5$ (see Example 21).



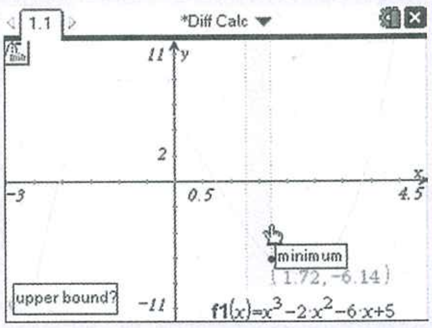
Press menu 6:Analyze Graph | 2:Minimum

Press enter

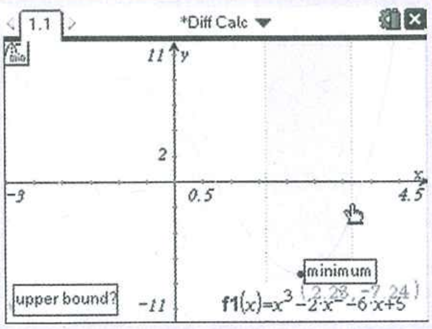
To find the minimum you need to give the lower and upper bounds of a region that includes the minimum.
The GDC shows a line and asks you to set the lower bound.
Move the line using the touchpad and choose a position to the left of the minimum.
Click the touchpad.



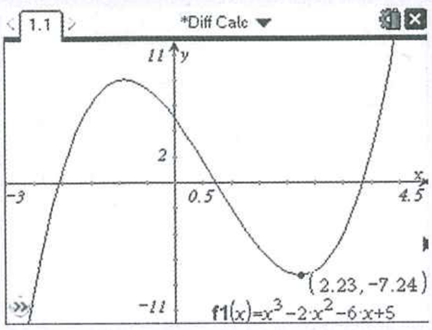
The GDC shows another line and asks you to set the upper bound.
Use the touchpad to move the line so that the region between the upper and lower bounds contains the minimum.
Note: The minimum point in the region that you have defined is being shown. In this screenshot it is not the local minimum point. Make sure you move the line beyond the point you are looking for.



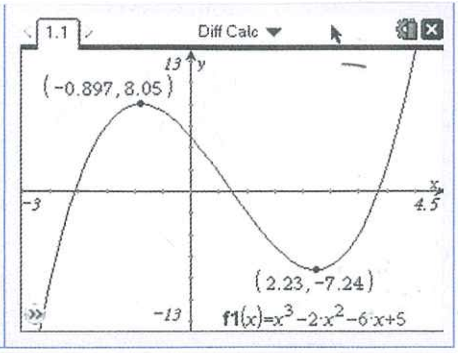
When the region contains the minimum, the GDC will display the word 'minimum' in a box and a point that lies between the lower and upper bounds. The point displayed is clearly between the upper and lower bounds.
Click the touchpad.



The GDC displays the local minimum at the point (2.23, −7.24).



Press menu 6:Analyze Graph | 3:Maximum to find the local maximum point on the curve in exactly the same way. The maximum point is $(-0.897, 8.05)$.



Derivatives

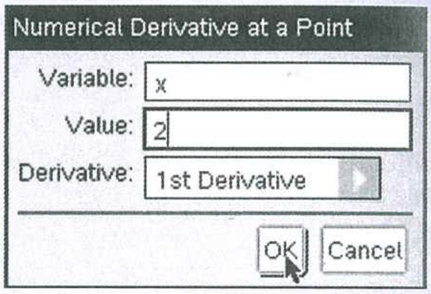
2.4 Finding a numerical derivative

Using the calculator it is possible to find the numerical value of any derivative for any value of x. The calculator will not, however, differentiate a function algebraically. This is equivalent to finding the gradient at a point graphically (see Section 2.1 example 21).

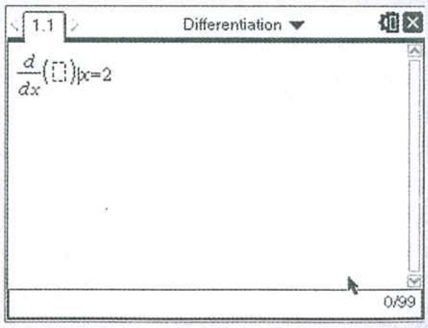
Example 24

If $y = \frac{x+3}{x}$, evaluate $\frac{dy}{dx}\Big|_{x=2}$

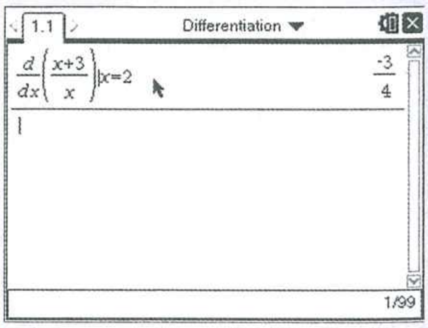
Open a new document and add a Calculator page.
Press menu 4:Calculus | 1:Numerical Derivative at a Point...
Leave the variable as x and the Derivative as 1st Derivative.
Change the Value to the value of x at which you wish to evaluate the derivative, in this case $x = 2$.



Enter the function in the template.
Press enter



The calculator shows that the value of the first derivative of $y = \left(\frac{x+3}{x}\right)$ is $-\frac{3}{4}$ when $x = 2$.



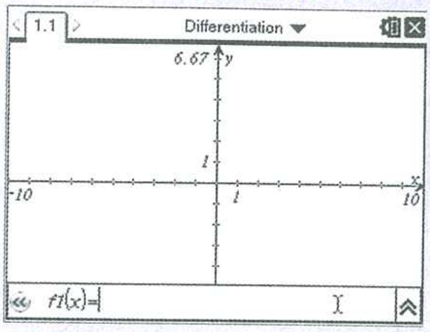
2.5 Graphing a numerical derivative

Although the calculator can only evaluate a numerical derivative at a point, it will graph the gradient function for all values of x.

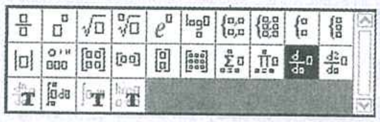
Example 25

If $y = \frac{x+3}{x}$, draw the graph of $\frac{dy}{dx}$.

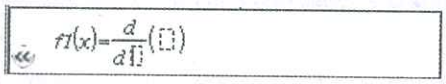
Open a new document and add a Graph page.
The entry line is displayed at the bottom of the work area.
The default graph type is Function, so the form "$f1(x)=$" is displayed.
The default axes are $-10 \le x \le 10$ and $-6.67 \le y \le 6.67$.



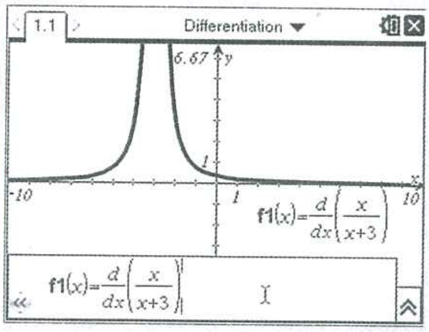
Press the templates button marked and choose the numerical derivative.



In the template enter x and the function $\frac{x+3}{x}$.
Press enter



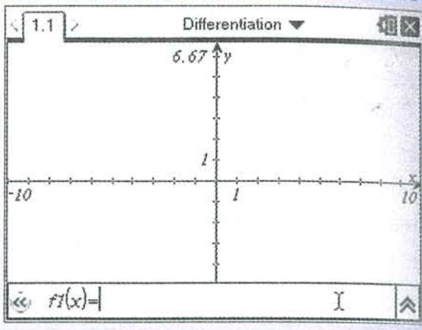
The calculator displays the graph of the numerical derivative function of $y = \frac{x+3}{x}$.



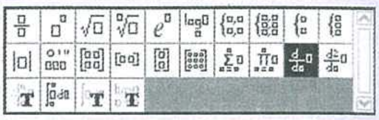
Example 26

Find the values of x on the curve $y = \frac{x^3}{3} + x^2 - 5x + 1$ where the gradient is 3.

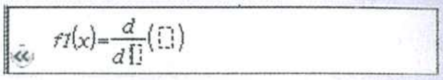
Open a new document and add a Graphs page.
The entry line is displayed at the bottom of the work area.
The default graph type is Function, so the form "$f1(x) =$" is displayed.
The default axes are $-10 \le x \le 10$ and $-6.67 \le y \le 6.67$.



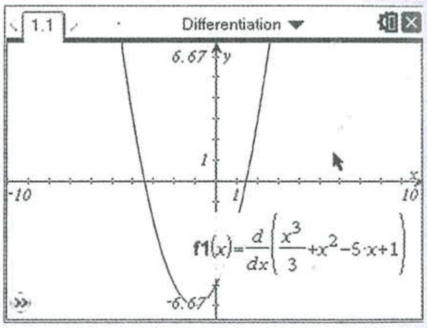
Press the templates button marked and choose the numerical derivative.



In the template enter x and the function $\frac{x^3}{3} + x^2 - 5x + 1$.
Press enter.



The calculator displays the graph of the numerical derivative function of $y = \frac{x^3}{3} + x^2 - 5x + 1$.

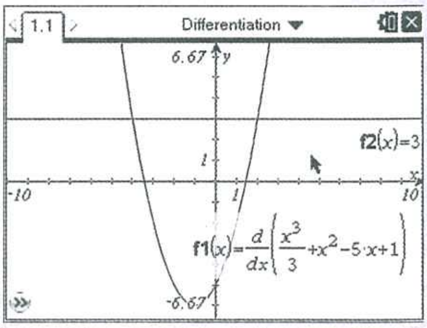


Using the touchpad, click on to open the entry line at the bottom of the work area.

Enter the function $f2(x) = 3$

Press enter

The calculator now displays the curve and the line $y = 3$.

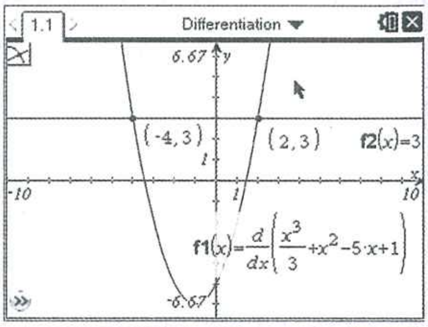


Press menu 7:Points & Lines | 3:Intersection Point(s)

Using the touchpad, select graph $f1$ and graph $f2$.

The calculator displays the coordinates of the intersection points of the gradient function and the line $y = 3$.

The curve has gradient 3 when $x = -4$ and $x = 2$



2.6 Using the second derivative

The calculator can find first and second derivatives. The second derivative can be used to determine whether a point is a maximum or minimum point.

Example 27

Find the stationary points on the curve $f(x) = x^4 - 4x^3$ and determine their nature.

$f(x) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2$

At stationary points

$f'(x) = 0$

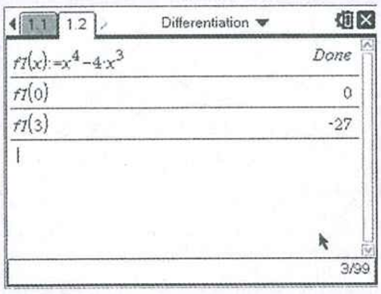
$4x^3 - 12x^2 = 0$

$4x^2 - (x - 3) = 0$

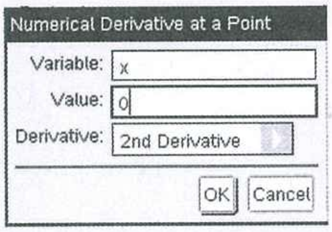
Therefore $x = 0$ or $x = 3$

Use the calculator to find the coordinates of the points and to determine their nature.
Open a new document and add a Calculator page.
Define the function $f1(x)$
Type F 1 (X) ctrl := and type the function.
Evaluate the function when $x = 0$ and $x = 3$

The stationary points are at (0, 0) and (3, –27)



Press menu 4:Calculus | 1:Numerical Derivative at a Point...
Leave the variable as x and choose 2nd Derivative. Change the Value to the value of x at which you wish to evaluate the derivative, in this case $x = 0$ (and $x = 3$).

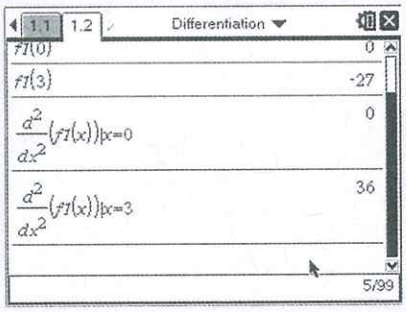


Enter $f1(x)$ in the template as the function.

Repeat for the second derivative when $x = 3$

(Note: you can cut and past the expression and change the 0 to 3)

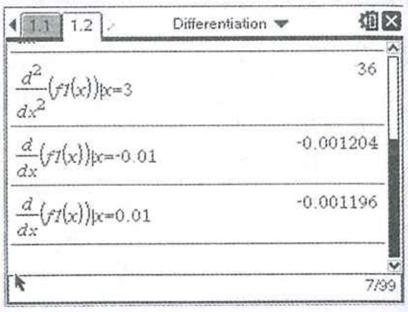
In this case we are not certain what the nature of the stationary point is at (0, 0) but the point (3, –27) is a minimum because $f''(x) > 0$



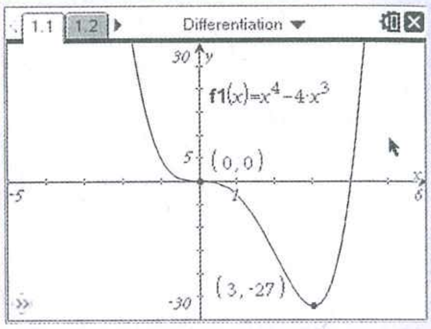
Evaluate $f'(x)$ either side of $x = 0$.

In this case using $x = -0.01$ and $x = 0.01$

The gradient is negative either side of the stationary point. Hence (0, 0) is a negative point of inflexion.



The graph on the right illustrates the curve, the minimum at (3, –27) and the point of inflexion at (0, 0).



3 Integral calculus

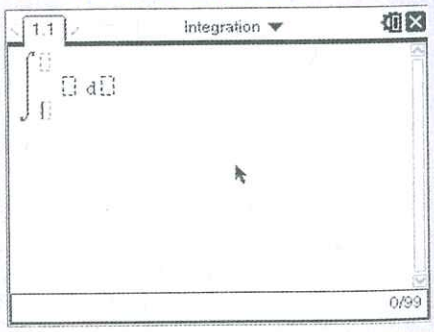
The calculator can find the values of definite integrals either on a calculator page or graphically. The calculator method is quicker, but the graphical method is clearer and shows discontinuities, negative areas and other anomalies that can arise.

3.1 Finding the value of an indefinite integral

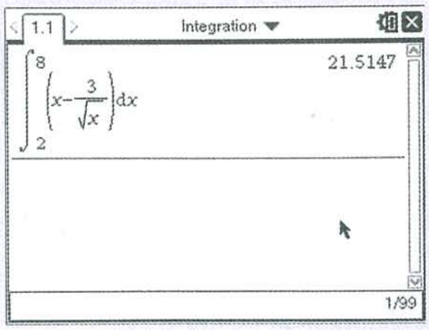
Example 28

Evaluate $\int\left(x-\frac{3}{\sqrt{x}}\right)\mathrm{d}x$

Open a new document and add a Calculator page.
Press menu 4:Calculus | 1:Numerical Integral...
Enter the upper and lower limits, the function and x in the template.
Use the ▼ ▲ ◀ ▶ keys to navigate around the template.
In this example you will also use templates to enter the rational function and the square root.



The value of the integral is 21.5 (to 3 sf)



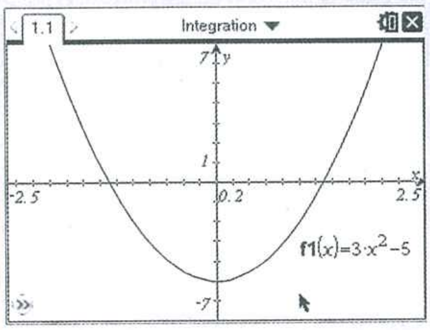
3.2 Finding the area under a curve

Example 29

Find the area bounded by the curve $y = 3x^2 - 5$, the x-axis and the lines $x = -1$ and $x = 1$.

Open a new document and add a Graphs page.
The entry line is displayed at the bottom of the work area.
The default graph type is Function, so the form "$f1(x)$=" is displayed.

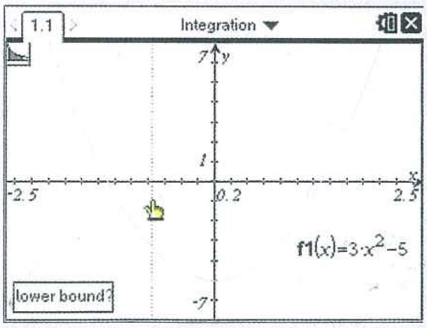
The default axes are $-10 \le x \le 10$ and $-6.67 \le y \le 6.67$.
Type the function $3x^2 - 5$
Press enter



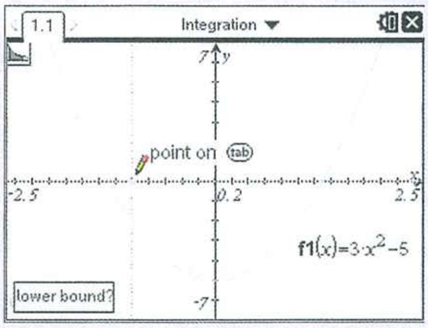
Press menu 6:Analyze Graph | 6:Integral

The calculator prompts you to enter the lower limit for the integral. There are several ways to do this.

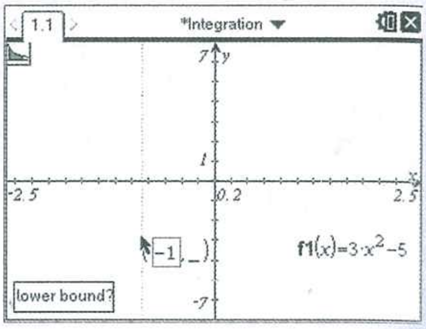
You can click manually. This is not very accurate, however, and you will need to add the coordinates of the point you entered and edit them to obtain an accurate figure.



You can use the points on the axis.
Here the scale was set to 0.2, so the point (–1, 0) can be selected as shown.



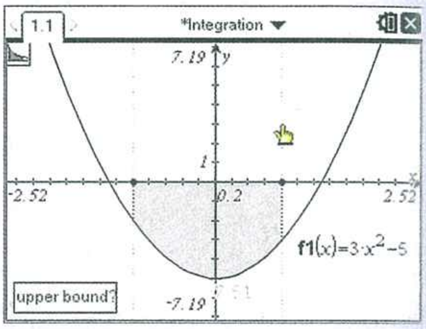
You can enter the point with the keyboard.
Enter a left bracket (and then type (-) 1 and press enter
There is no need to complete the coordinates.



Repeat for the upper limit.

The calculator displays a changing value for the area.

Using one of the methods above, select a point where the value of x is 1.



Repeat for the upper limit.

The area found is shaded and the value of the integral (–8) is shown on the screen.

Note: since the area lies below the x-axis in this case, the integral is negative.

The required area is 8.