2019 Differential and Integral Calculus Assignment Markscheme

1.

(a) valid approach R1

e.g. $f''(x) = 0$, the max and min of f' gives the points of inflexion on f

–0.114, 0.364 (accept (–0.114, 0.811) and (0.364, 2.13)) A1A1 N1N
1

(b) **METHOD 1**

graph of g is a quadratic function R1 N1
a quadratic function does not have any points of inflexion R1 N1

METHOD 2

graph of g is concave down over entire domain R1 N1
therefore no change in concavity R1 N1

METHOD 3

$g''(x) = -144$ R1 N1
therefore no points of inflexion as $g''(x) \neq 0$ R1 N1

[5]

2.

(a) (i) –1.15, 1.15 A1A1 N2

(ii) recognizing that it occurs at P and Q (M1)

e.g. $x = -1.15$, $x = 1.15$

$k = -1.13$, $k = 1.13$ A1A1 N3

(b) evidence of choosing the product rule (M1)

e.g. $uv' + vu'$

derivative of x^3 is $3x^2$ (A1)

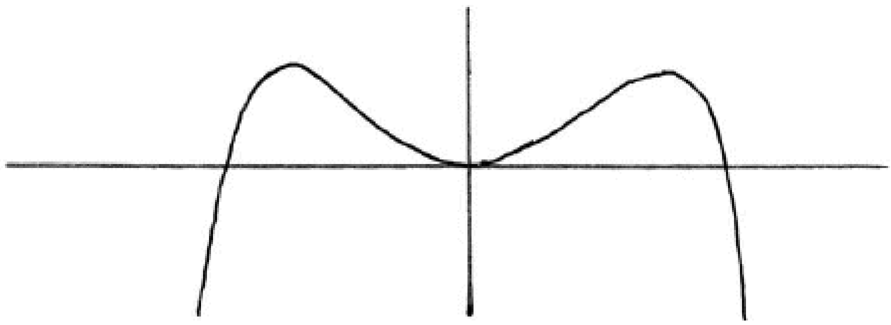
derivative of $\ln(4 - x^2)$ is $\frac{-2x}{4-x^2}$ (A1)

correct substitution A1

e.g. $x^3 \times \frac{-2x}{4-x^2} + \ln(4-x^2) \times 3x^2$

$g'(x) = \frac{-2x^4}{4-x^2} + 3x^2 \ln(4-x^2)$ AG N0

(c)



A1A1 N2

(d) $w = 2.69$, $w < 0$ A1A2 N2

[14]

3.

(a) 2.31 A1 N1

(b) (i) 1.02 A1 N1

(ii) 2.59 A1 N1

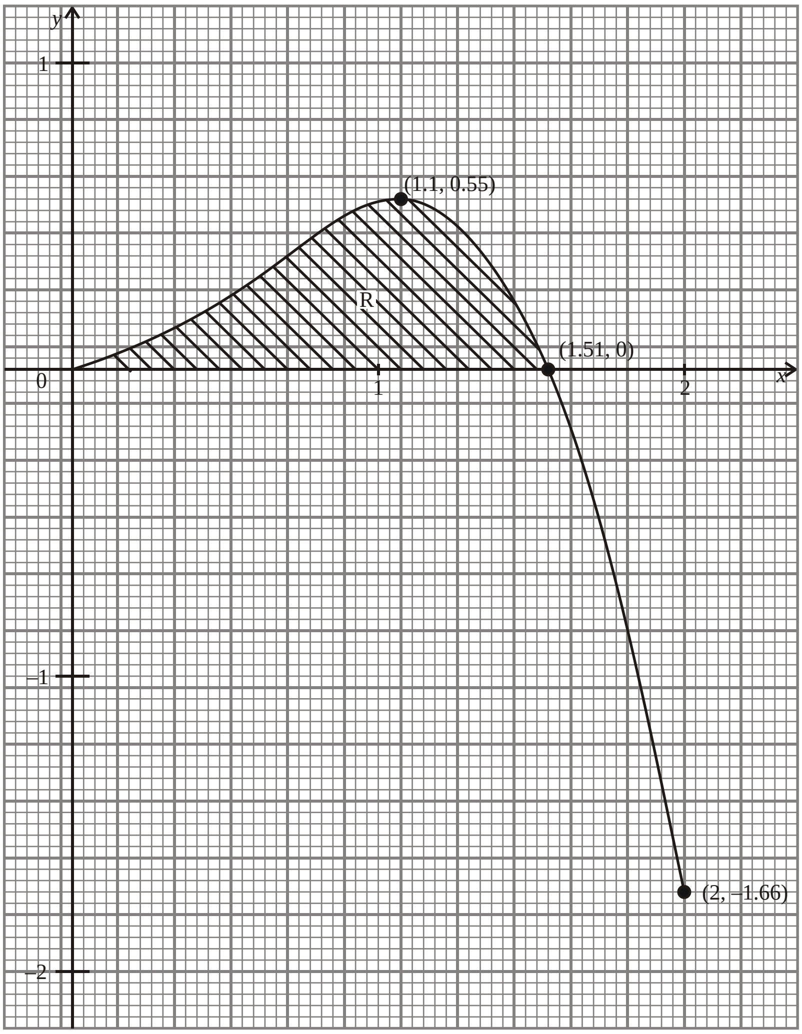
(c) $\int_p^q f(x)\mathrm{d}x = 9.96$ A1 N1

split into two regions, make the area below the x-axis positive R1R1 N2

[6]

4.

(a)(i) & (c)(i)



(A3)

***Notes:** The sketch does **not** need to be on graph paper. It should have the correct shape, and the points (0, 0), (1.1, 0.55), (1.57, 0) and (2, –1.66) should be indicated in some way.*
Award (A1) for the correct shape.
Award (A2) for 3 or 4 correctly indicated points, (A1) for 1 or 2 points.

(ii) Approximate positions are
positive x-intercept (1.57, 0) (A1)
maximum point (1.1, 0.55) (A1)
end points (0, 0) and (2, –1.66) (A1)(A1) 7

(b) $x^2 \cos x = 0$ $\quad x \neq 0 \Rightarrow \cos x = 0$ (M1)

$\Rightarrow x = \dfrac{\pi}{2}$ (A1) 2

***Note:** Award (A2) if answer correct.*

(c) (i) see graph (A1)

(ii) $\int_0^{\frac{\pi}{2}} x^2 \cos x \, dx$ (A2) 3

Note: *Award (A1) for limits, (A1) for rest of integral correct (do not penalize missing dx).*

(d) Integral = 0.467 (G3)

OR

Integral = $\left[x^2 \sin x + 2x \cos x - 2 \sin x\right]_0^{\pi/2}$ (M1)

$= \left[\frac{\pi^2}{4}(1) + 2\left(\frac{\pi}{2}\right)(0) - 2(1)\right] - [0 + 0 - 0]$ (M1)

$= \frac{\pi}{2} - 2$ (exact) **or** 0.467 (3 sf) (A1) 3

[15]

5.

(a) evidence of valid approach (M1)

e.g. $f(x) = 0$, graph

$a = -1.73$, $b = 1.73$ $(a = -\sqrt{3}, b = \sqrt{3})$ A1A1 N3

(b) attempt to find max (M1)

e.g. setting $f'(x) = 0$, graph

$c = 1.15$ (accept (1.15, 1.13)) A1 N2

(c) attempt to substitute either limits or the function into formula M1

e.g. $V = \pi\int_0^c [f(x)]^2 dx, \pi\int \left[x \ln(4 - x^2)\right]^2, \pi\int_0^{1.149...} y^2 dx$

$V = 2.16$ A2 N2

(d) valid approach recognizing 2 regions (M1)

e.g. finding 2 areas

correct working (A1)

e.g. $\int_0^{-1.73...} f(x)dx + \int_0^{1.149...} f(x)dx; -\int_{-1.73...}^{0} f(x)dx + \int_0^{1.149...} f(x)dx$

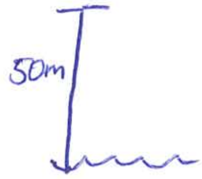
area = 2.07 (accept 2.06) A2 N3

[12]

Question 6 (12 marks)

Amy does a bungee jump from a platform 50 m above a river. Let h be her height above the river, in metres, at a time ty seconds after jumping. Her velocity is given by $v = 2t^2 - 10t$.

a) What is the initial acceleration that Amy experiences?



$a = \frac{dv}{dt} = 4t - 10$ (1M)

at $t=0$, $a = 4\times 0 - 10$

$= -10 \text{ ms}^{-2}$ (1A) (decelerating at 10 ms^{-2})

(2)

b) At what time is Amy's velocity zero?

$2t^2 - 10t = 0$ (1M)

$2t(t-5) = 0$

$t = 0$ or 5 secs (1A)

(2)

c) How close to the river does Amy get?

Bungee stops when $v = 0$.

$s = \int v\, dt$

$= \int 2t^2 - 10t\, dt$

$= \frac{2t^3}{3} - \frac{10t^2}{2} + c$ (1M)

When $t=0$, $s=0$, so $c=0$.

$s = \frac{2}{3}t^3 - 5t^2$

at $t=5$,

$s = \frac{2}{3}\times 5^3 - 5\times 5^2$

$= -41\frac{2}{3}$ m. (1A)

$\therefore$ height above river $= 50 - 41\frac{2}{3}$

$= 8\frac{1}{3}$ m. (1A)

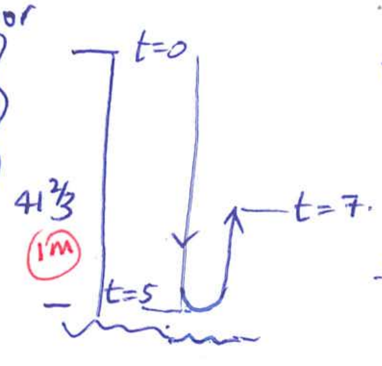
(3)

d) What distance does Amy travel in the first seven seconds?

dist $= \int_0^7 |2t^2 - 10t|\, dt$ (1M) (1M)

$= 67$ m (1A)

or



$t=7$, $s = \frac{2\times 7^3}{3} - 5\times 7^2$

$= -16\frac{1}{3}$. (1M)

$\therefore$ total $= 41\frac{2}{3} + 25\frac{1}{3}$

$= 67$ m. (1A)

(3)

e) How long does it take for Amy to return to the platform?

returns when $s = 0$.

$\frac{2}{3}t^3 - 5t^2 = 0$ (1M)

$t^2(\frac{2}{3}t - 5) = 0$

$t = 0$ or $\frac{2}{3}t - 5 = 0$.

$t = 7\frac{1}{2}$.

$\therefore$ returns after $7\frac{1}{2}$ sec (1A)

(2)

Question 7 (14 marks)

A boat travelling in a straight line has its engine turned off at $t = 0$. Its velocity at time t seconds thereafter is given by $v(t) = \dfrac{100}{(t+2)^2}$ ms $^{-1}$.

a) Find the initial velocity of the boat, and its velocity after 3 seconds.

$v(0) = \frac{100}{4} = 25 ms^{-1}$ (1A)

$v(3) = \frac{100}{25} = 4 ms^{-1}$ (1A)

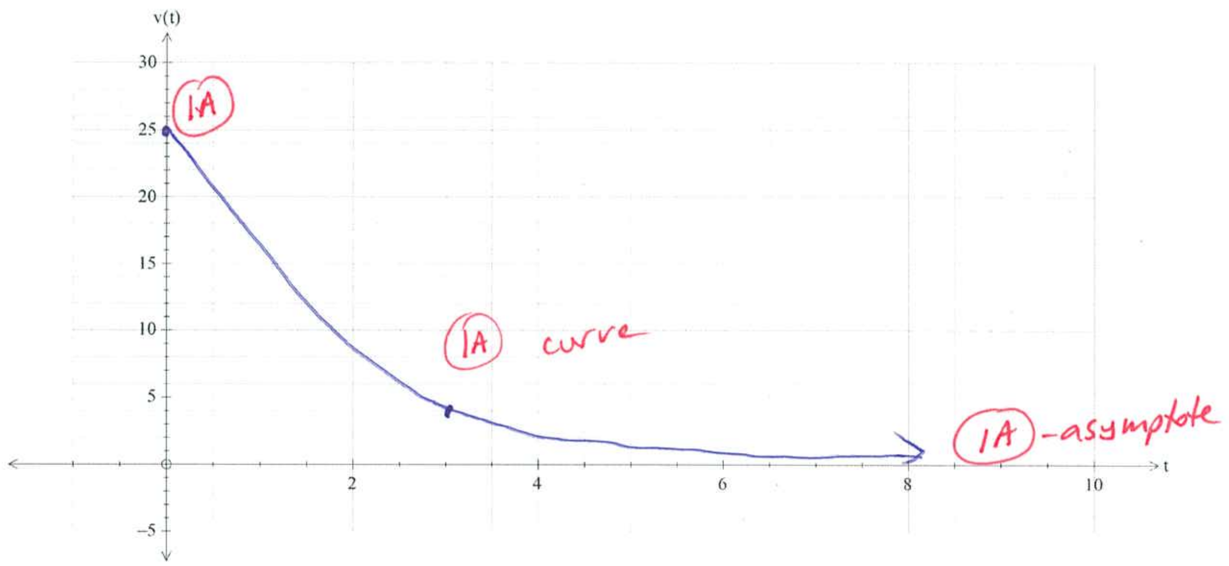
(2)

b) Discuss $v(t)$ as $t \to \infty$.

As $t \to \infty$, $v(t) \to 0$ or, decreasing function, asymptote at $v(t) = 0$. (1A)

(1)

c) Sketch a graph of $v(t)$ against t.



(3)

d) Find how long it takes for the boat to travel 30 metres from when the engine is turned off.

$s = \int v(t)\, dt$

$= \int 100(t+2)^{-2}\, dt$

$= 100 \frac{(t+2)^{-1}}{-1} + c$ (1M)

at $t=0$, $s=0$,

$0 = \frac{-100}{2} + c$

$c = 50$

$\therefore s = \frac{-100}{t+2} + 50$ (1A)

when $s = 30$

$30 = \frac{-100}{t+2} + 50$ (1M)

$\frac{100}{t+2} = 20$

$5 = t + 2$

$t = 3.$ (1A)

$\therefore$ after 3 sec, boat has travelled 30m.

(4)

e) Find the acceleration of the boat at any time t.

$$a = \frac{dv}{dt} \quad \text{(1M)}$$

$$= 100 \times -2(t+2)^{-3}$$

$$= \frac{-200}{(t+2)^3} \quad \text{(1A)}$$

(2)

f) Show that $\frac{dv}{dt} = -kv^{\frac{3}{2}}$, and find the value of the constant k.

$$\frac{-200}{(t+2)^3} = -k\left(\frac{100}{(t+2)^2}\right)^{3/2}$$

$$= -k \times 100^{3/2} \times \left[(t+2)^{-2}\right]^{3/2}$$

$$= -k \times 1000 \times (t+2)^{-3}$$

$$\frac{-200}{(t+2)^3} = \frac{-1000k}{(t+2)^3} \quad \text{(1M)}$$

(2)

$$\therefore -200 = -1000k$$

$$k = \frac{-200}{-1000}$$

$$= \frac{1}{5} \quad \text{(1A)}$$