St Leonard's College

Year 12 IB Mathematics SL

Integral Calculus Test – Calculator

Date: 30 April 2019

Start Time: 12:15

Finish Time: 12:55

Total Time Allowed for Task: 40 minutes

Student Name:_____

Teacher Name: _____

Conditions

Calculators allowed.

Data booklet permitted.

All answers should be given **exactly** or correct to **three** significant figures unless otherwise specified.

Results

Overall: **/41** **%** **(1-7 Grade)**

1a. Let $f(x) = (x-1)(x-4)$.

Find the x-intercepts of the graph of f.

[3 marks]

1b. The region enclosed by the graph of f and the *x*-axis is rotated 360° about the *x*-axis.

Find the volume of the solid formed.

[3 marks]

2a. Let $f(x) = \sqrt[3]{x^4} - \frac{1}{2}$.

Find $f'(x)$.

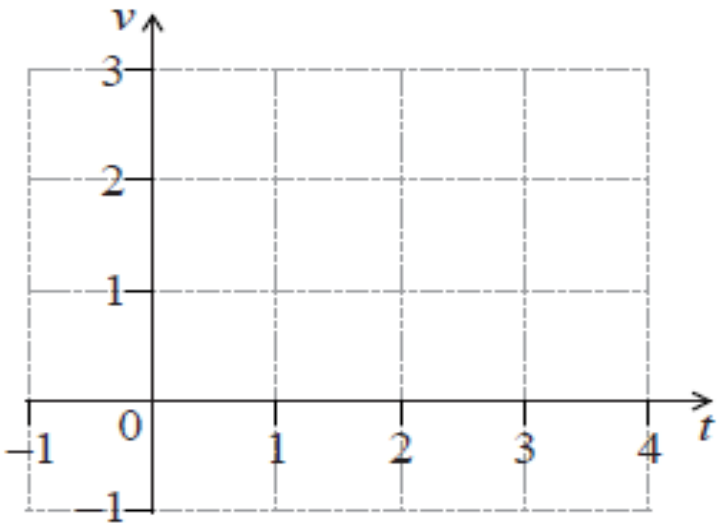
[2 marks]

2b. Find $\int f(x)\mathrm{d}x$.

[4 marks]

3a. A particle moves along a straight line such that its velocity, v ms^{-1}, is given by $v(t) = 10te^{-1.7t}$, for $t \geqslant 0$.

On the grid below, sketch the graph of v, for $0 \leqslant t \leqslant 4$.



[3 marks]

3b. Find the distance travelled by the particle in the first three seconds.

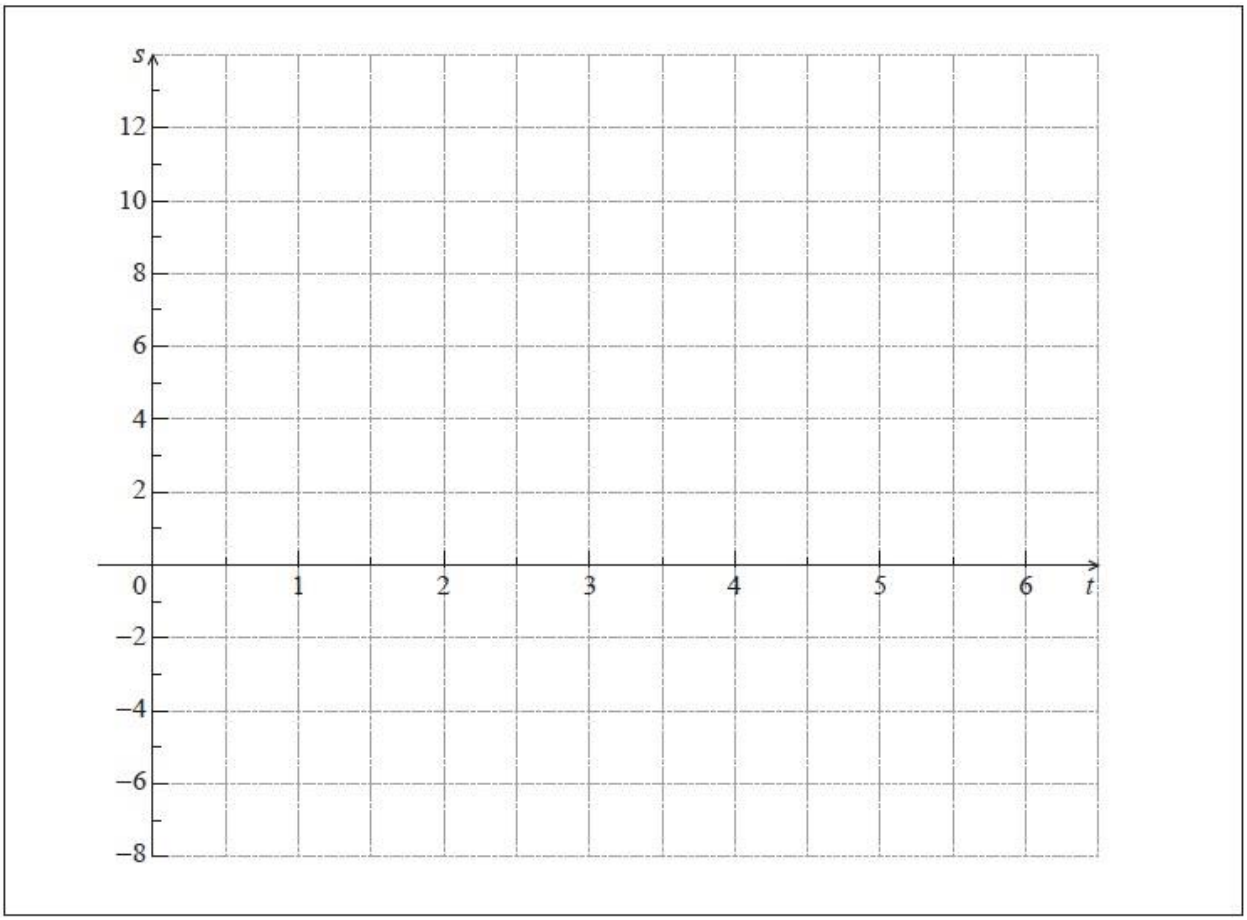
[2 marks]

3c. Find the velocity of the particle when its acceleration is zero.

[3 marks]

4a. A particle's displacement, in metres, is given by $s(t) = 2t\cos t$, for $0 \leq t \leq 6$, where *t* is the time in seconds.

On the grid below, sketch the graph of s.

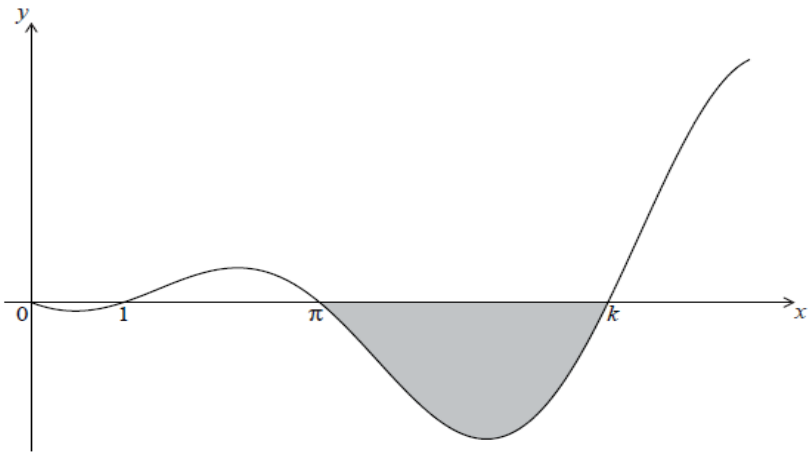


[4 marks]

4b. Find the maximum velocity of the particle.

[3 marks]

5a. The graph of $y = (x - 1)\sin x$, for $0 \leq x \leq \frac{5\pi}{2}$, is shown below.



The graph has x-intercepts at 0, 1, π and k.

Find k.

[2 marks]

5b. The shaded region is rotated $360°$ about the x-axis. Let V be the volume of the solid formed.

Write down an expression for V.

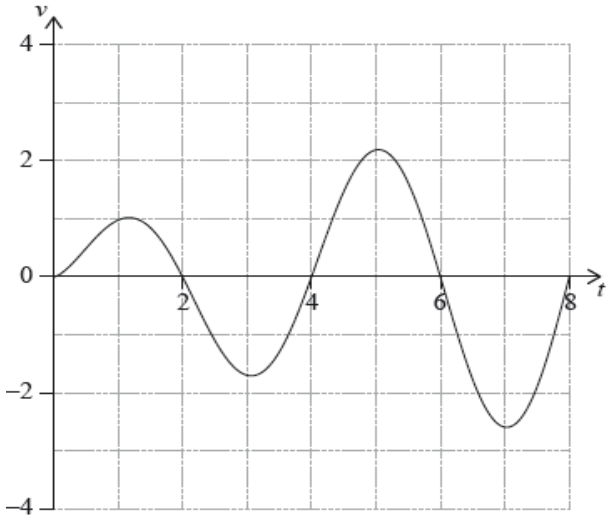
[3 marks]

5c. The shaded region is rotated $360°$ about the x-axis. Let V be the volume of the solid formed.

Find V.

[2 marks]

6a. A particle P moves along a straight line. Its velocity v_{P} m s^{-1} after t seconds is given by $v_{\text{P}} = \sqrt{t}\sin\left(\frac{\pi}{2}t\right)$, for $0 \leqslant t \leqslant 8$. The following diagram shows the graph of v_{P}.



Write down the first value of t at which P changes direction.

[1 mark]

6b. Find the **total** distance travelled by P, for $0 \leqslant t \leqslant 8$.

[2 marks]

6c. A second particle Q also moves along a straight line. Its velocity, v_{Q} m s^{-1} after t seconds is given by $v_{\text{Q}} = \sqrt{t}$ for $0 \leqslant t \leqslant 8$. After k seconds Q has travelled the same total distance as P.

Find k.

[4 marks]