Multiple-choice questions

1  In the sequence 5, 14, 23, 32, … the value of the common difference, \( d \), is:
   A  –9
   B  9
   C  19
   D  27
   E  41

2  The twelfth term, \( t_{12} \), in the sequence 43, 37, 31, 25, … is:
   A  –72
   B  –23
   C  –6
   D  12
   E  19

3  Using the recurrence relation \( t_1 = 7, \ t_{n+1} = t_n + 6 \), the fifth term would be:
   A  11
   B  13
   C  18
   D  25
   E  31

4  A bicycle costs $50 to hire for the first day and $40 for each extra day. If \( C_n \) is the cost of hiring the bicycle for \( n \) days, the recurrence relation is:
   A  \( C_0 = 50, \ C_{n+1} = C_n + 40 \)
   B  \( C_0 = 40, \ C_n = C_{n+1} + 50 \)
   C  \( C_1 = 50, \ C_n = C_{n+1} + 40 \)
   D  \( C_0 = 40, \ C_{n+1} = C_n + 50 \)
   E  \( C_1 = 50, \ C_{n+1} = C_n + 4 \)
5  The common ratio, \( r \), of the sequence 81, 54, 36, 24, \( \ldots \) is:
   A  \(-57\)
   B  \(\frac{2}{3}\)
   C  \(\frac{3}{4}\)
   D  \(\frac{5}{2}\)
   E  57

6  The difference equation for the sequence 5, 20, 80, \( \ldots \) is:
   A  \(t_1 = 5, t_{n+1} = 5t_n\)
   B  \(t_1 = 5, t_{n+1} = 4t_n\)
   C  \(t_1 = 4, t_{n+1} = 5t_n\)
   D  \(t_1 = 5, t_{n+1} = t_n + 15\)
   E  \(t_1 = 4, t_{n+1} = t_n + 5\)

7  In the sequence of patterns below pattern 1 used eight sticks.

![Patterns](image)

Pattern 1  Pattern 2  Pattern 3

The number of sticks required to make Pattern 100 would be:
   A  503
   B  508
   C  795
   D  800
   E  805

8  A 9\% decrease is made by using a common ratio of:
   A  \(-109\)
   B  \(-9\)
   C  \(-1.09\)
   D  \(-0.91\)
   E  0.91
9 The number of feral cats in a wildlife reserve increases by 15% each year. If there were originally 200 feral cats in the reserve, the recurrence relation for the number of feral cats \( C_n \) after \( n \) years is:

A \( C_1 = 200, C_{n+1} = 1.15C_n \)

B \( C_1 = 200, C_{n+1} = 0.15C_n \)

C \( C_1 = 200, C_{n+1} = 15C_n \)

D \( C_0 = 200, C_n = 1.15C_{n+1} \)

E \( C_0 = 200, C_{n+1} = 1.15C_n \)

10 The following recurrence relation can be used to model a compound interest investment of $3000, paying interest at the rate of 11% per annum.

\[
V_0 = 2000, \quad V_{n+1} = 1.09V_n
\]

After how many years will the value of the investment first exceed $6000?

A 7

B 8

C 9

D 10

E 11

11 In a Fibonacci type sequence \( t_5 = 24 \) and \( t_6 = 39 \). The value for \( t_8 \) is:

A 21

B 54

C 63

D 69

E 102

12 The first three terms in the sequence using the recurrence relation \( t_1 = 5, t_{n+1} = 3t_n - 4 \), are:

A \(-1, 2, 5, \ldots\)

B \(3, -4, 5, \ldots\)

C \(5, 3, -4, \ldots\)

D \(5, 11, 29, \ldots\)

E \(11, 29, 83, \ldots\)
Extended-response questions

1 Jessica accepted a job with a salary of $50 000 in the first year and an increase of $6000 in each following year.
   a Show her salary for each of the first 10 years in a table.
   b Would the points on a graph of the table of values be a straight line or a curve?
      Give an explanation to support your answer.
Rebecca accepted a job which paid $44 000 in the first year and increased by 12% in each following year.
   c Show her salary for each of the first 10 years in a table.
   d Would the points on a graph of the table of values be a straight line or a curve?
      Give an explanation to support your answer.

2 An oil well started producing 1000 barrels of oil per day. The rate at which oil production reduces each day is called the decline rate and is used to predict the productive life of the oil well.
   a Output for the first three days was recorded as:
      1000, 980, 960.4
      The decline rate is the percentage that production is reducing by each day. Calculate the decline rate for this oil well.
      b State the recurrence relation for the oil production $P_n$ on the $n^{th}$ day of production.
   c What is the expected production for the well on the tenth day of operation?
   d How many days of production will be possible before the daily output falls below 700 barrels?
Chapter 8 test 2 answers

Multiple-choice questions

1  B
2  B
3  E
4  E
5  B
6  B
7  A
8  E
9  E
10 A
11 A
12 D

Extended-response questions

1  a

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<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<td>Salary</td>
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<td>$86 000</td>
<td>$92 000</td>
<td>$98 000</td>
<td>$104 000</td>
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</table>

b  A straight line. An arithmetic sequence produces a linear graph because the values increase or decrease by a fixed amount. In this case the values increase by $7000.

c

<table>
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<th>Year</th>
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<th>3</th>
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<td>$49 280</td>
<td>$55 193.60</td>
<td>$61 816.83</td>
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<tr>
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<td>$77 543.03</td>
<td>$86 848.20</td>
<td>$97 269.98</td>
<td>$108 942.38</td>
<td>$122 015.47</td>
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</table>
d  The graph would be a curve because the values do not increase by a constant amount.

2  a  2%
   b  $P_1 = 1000, \ P_{n+1} = 0.98P_n$
   c  833.75 barrels
   d  18 days