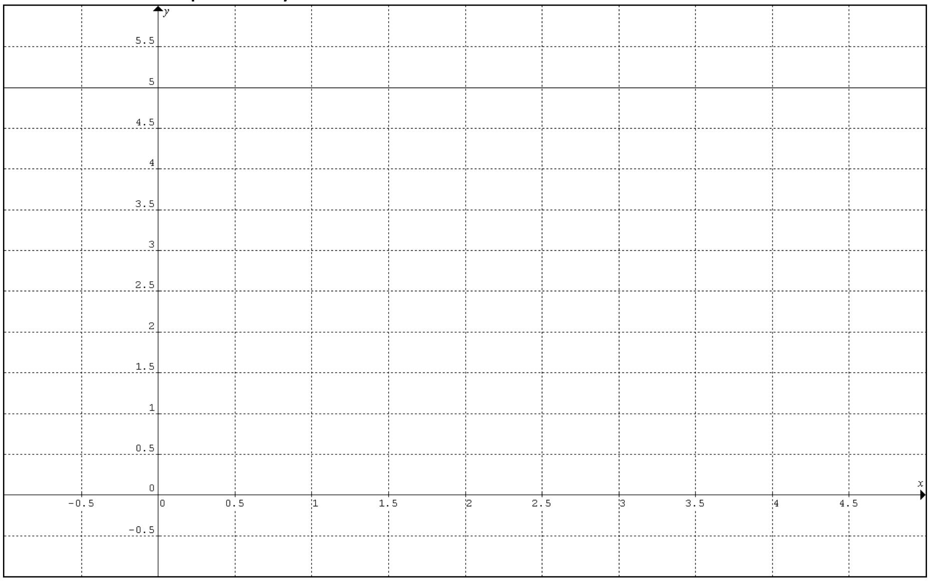
MM 3/4 Integral Calculus
Geometric Meaning of the Integral

St Leonard's College

Investigations

1. Consider the 'curve' with equation y = 5

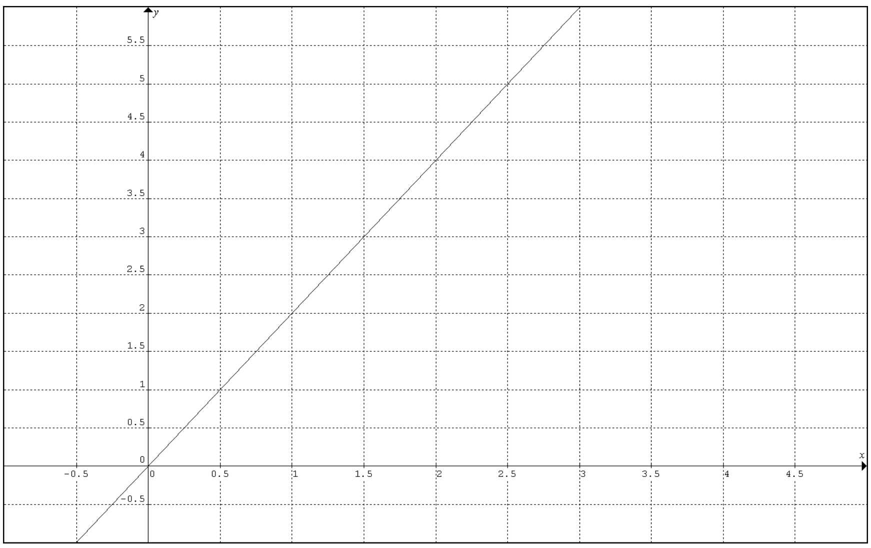


Now let's consider the area 'under' this curve successively from x = 0 to x = 4

From x = 0 to x =	Area, A
1	
2	
3	
4	

What does the rule for A appear to be?

2. Consider another simple 'curve' such as y = 2x



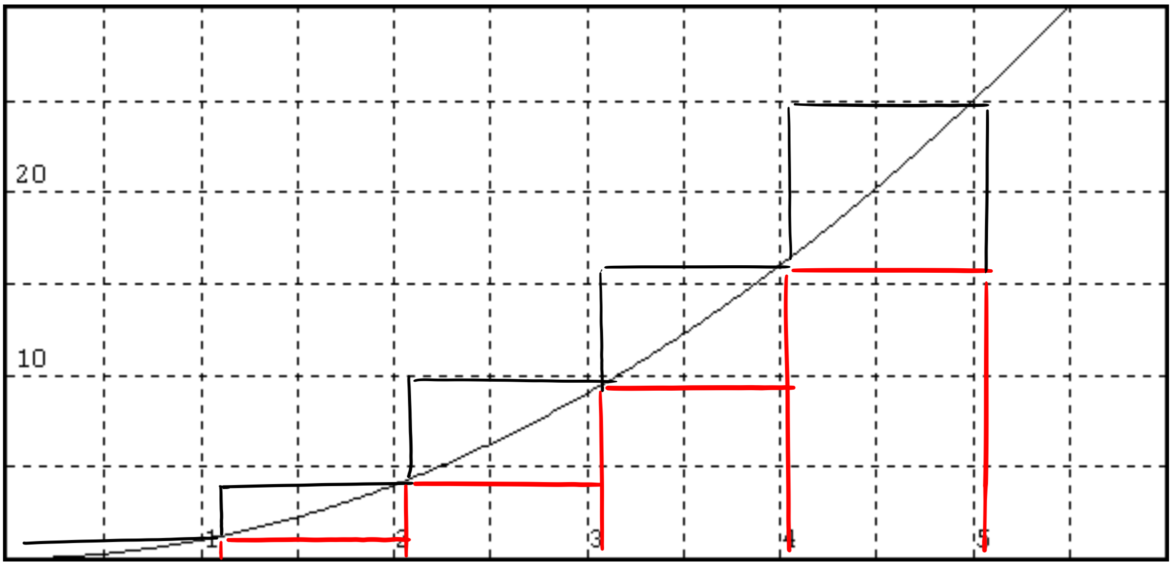
Now let's consider the area 'under' this curve successively from x = 0 to x = 4

From x = 0 to x =	Area, A
1	
2	
3	
4	

What does the rule for A appear to be?

3. Consider the graph below of the curve with equation $y = x^2$

We can't work out the area exactly this time!!
We can, however, work out an estimate of the area under the curve using 'right rectangles' (black) and 'left rectangles' (red).
Firstly, we partition the area into vertical strips as shown between x = 0 (lower bound) and x = 5 (upper bound)



NOTE : the height of the rectangles is given by the y value of the point on the curve where the rectangle touches the curve. The "left rectangles" heights are determined by the x value at the left of the rectangles and the "right rectangles" are given by the right ends of the rectangles.

x	Left rectangle area	Right rectangle area	x = 0 to x =	Area under curve is between	
0 to 1			1		
1 to 2			2		
2 to 3			3		
3 to 4			4		
4 to 5			5		

The Left Endpoint Estimate is given by the combined area of the Left Rectangles.

The Right Endpoint Estimate is given by the combined area of the Right Rectangles

What would make the estimated area values more accurate?

Using the CAS Graphing Calculator

Examples

(1) Finding the left endpoint estimate for the above function, f(x) = y = x^2.

The left endpoint estimate uses the y values for the x values on the left of each partition. ie the y values for x = 1, 2, 3, 4. In function notation this will be f(1), f(2), f(3), f(4)
The area of each rectangle is the width of the partition (in this case, 1) $\times$ the height (the y value)
(symbol: δx)

So the total area = 1$\times$f(1) + 1 $\times$ f(2) + 1$\times$ f(3) + 1 $\times$ f(4)
which can be written as $\sum_{x=1}^{4}(1 \times f(x))$

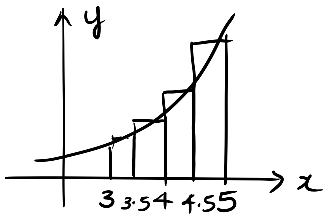
On the calculator :

Step 1: Define f(x)
Step 2: We can evaluate this sum using the template above the x sign

Your turn!! Use the calculator to find the right endpoint estimate of the area.

(2) Find the right endpoint estimate for the function f(x) = e^x for x $\in$ [3,5] using 4 partitions.

So...if there are to be 4 rectangles, these must be 0.5 in width...ie. $\delta x = 0.5$



Consider the graph at right.
The height of the right rectangles
will be given by f(3.5), f(4), f(4.5) and f(5)

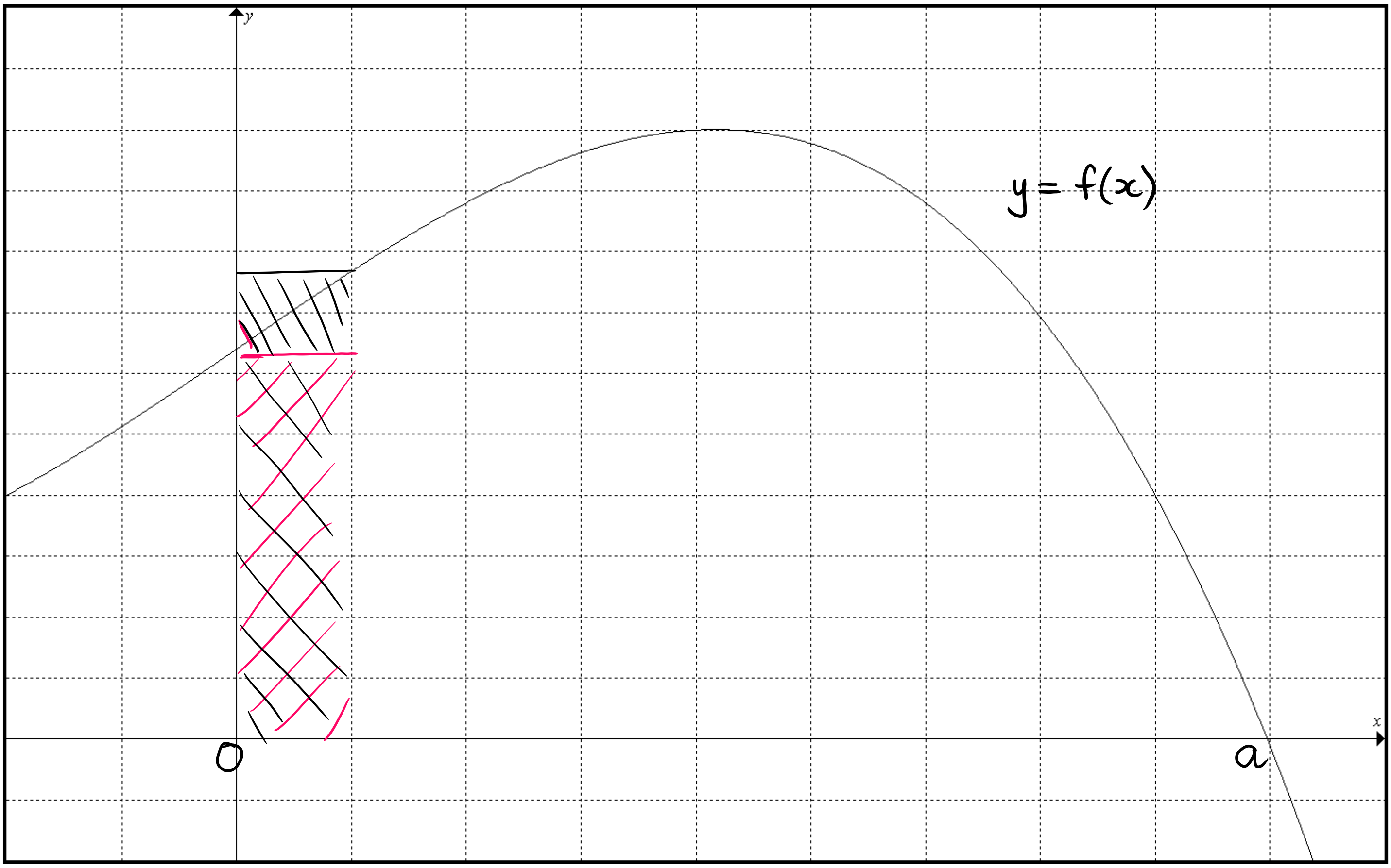
Total area = 0.5$\times$f(3.5) + 0.5$\times$f(4) +0.5$\times$f(4.5) +0.5$\times$f(5) = Σ 0.5$\times$f(k)   where k is 3.5, 4, 4.5, 5

On the calculator:
Step 1: Define f(x)
Step 2: We can evaluate this sum using $\sum_{k=7}^{10}(0.5 \times f(0.5k))$

Your turn!! Use the calculator

(a) to find the left endpoint and right endpoint estimates of the area under the curve with equation f(x) = $\frac{1}{x}$ for x $\in$[2,6] using 4 partitions. (b) to find the left endpoint and right endpoint estimates of the area under the curve with equation f(x) = $\log_e x$ x $\in$[3,6] using 30 partitions.

In general



True area under the curve will be somewhere between the left endpoint estimate and the right endpoint estimate. The smaller the width of the partitions (δx), the closer these values will be to each other, and the true area.

Ie. Area = $\lim_{\delta x \to 0} \sum_{i=1}^{n} \delta x_i \times f(x_i)$ = $\int f(x)dx$